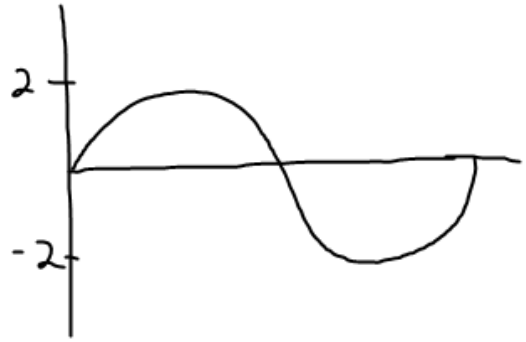
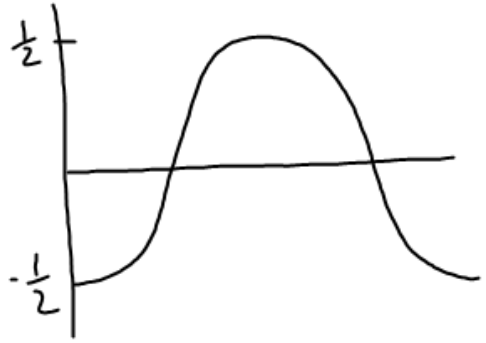
Vertical Stretch (a)

$y = a \sin(x)$

$y = 2\sin(x)$ → the graph will be stretched by 2 vertically



$y = -\frac{1}{2}\cos(x)$ → the negative flips the graph
the $\frac{1}{2}$ compresses it vertically



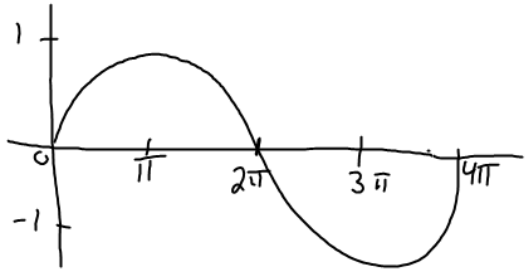
horizontal stretch/compression (b)

$y = \sin bx$

frequency $= \frac{1}{2}$

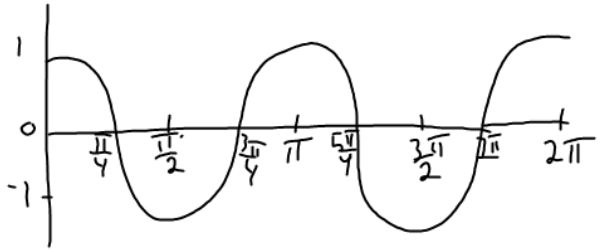
$y = \sin \frac{1}{2}x$ → will do $\frac{1}{2}$ cycle in 2π

Period $= \frac{2\pi}{b} \Rightarrow \frac{2\pi}{\frac{1}{2}} \Rightarrow 2\pi \cdot \frac{2}{1} = 4\pi$



$y = \cos 2x$ frequency is 2 so it will do 2 cycles in 2π

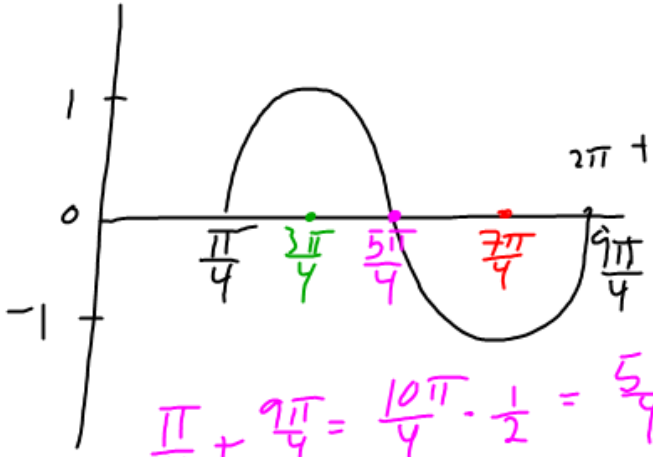
Period $= \frac{2\pi}{2} = \pi$



horizontal shift (c)

$y = \sin(x - c)$

$y = \sin(x - \frac{\pi}{4})$ shifts right $\frac{\pi}{4}$, gives you the starting endpoint



$2\pi + \frac{\pi}{4} \rightarrow \frac{8\pi}{4} + \frac{\pi}{4} = \frac{9\pi}{4}$

$\frac{\pi}{4} + \frac{9\pi}{4} = \frac{10\pi}{4} \cdot \frac{1}{2} = \frac{5\pi}{4}$

$\frac{\pi}{4} + \frac{5\pi}{4} = \frac{6\pi}{4} \cdot \frac{1}{2} = \frac{3\pi}{4}$

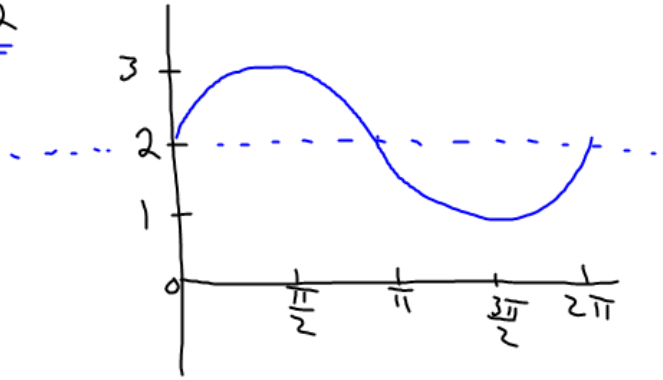
$\frac{5\pi}{4} + \frac{9\pi}{4} = \frac{14\pi}{4} \cdot \frac{1}{2} = \frac{7\pi}{4}$

Vertical shift (d)

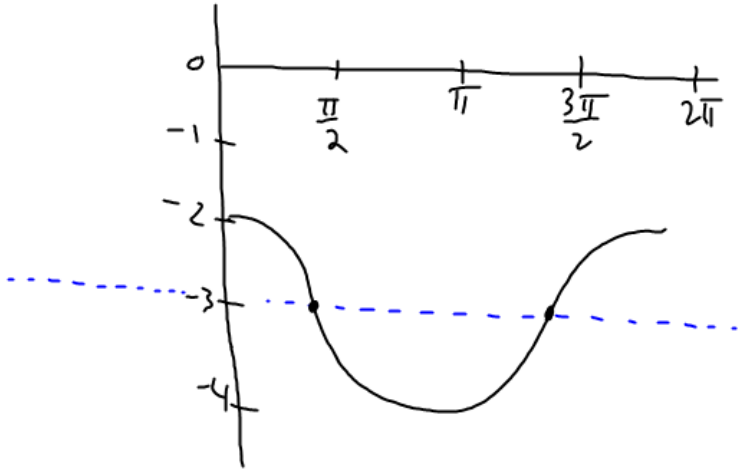
$y = \sin(x) + d$

The d-value gives you the mid-line

$y = \sin(x) + 2$

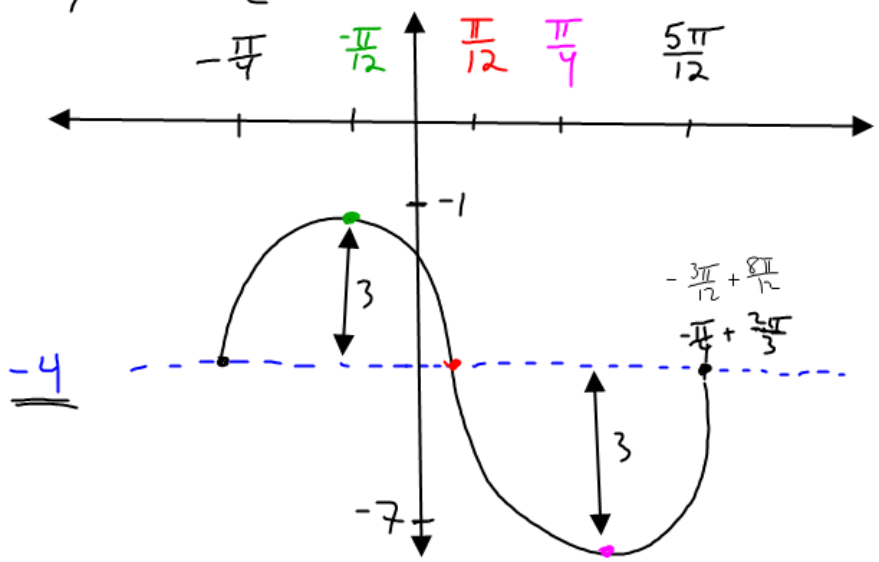


$y = -3 + \cos(x) \rightarrow y = \cos(x) - 3$



Example

$y = \overset{a}{3}\sin\left[\overset{b}{3}\left(x + \overset{c}{\frac{3\pi}{12}}\right)\right] \overset{d}{-4}$



$-\frac{\pi}{4} + \frac{5\pi}{12} = \frac{-3\pi}{12} + \frac{5\pi}{12} = \frac{2\pi}{12} \cdot \frac{1}{2} > \frac{\pi}{12}$

$-\frac{\pi}{4} + \frac{\pi}{12} = \frac{-3\pi}{12} + \frac{\pi}{12} = \frac{-2\pi}{12} \cdot \frac{1}{2} = -\frac{\pi}{12}$

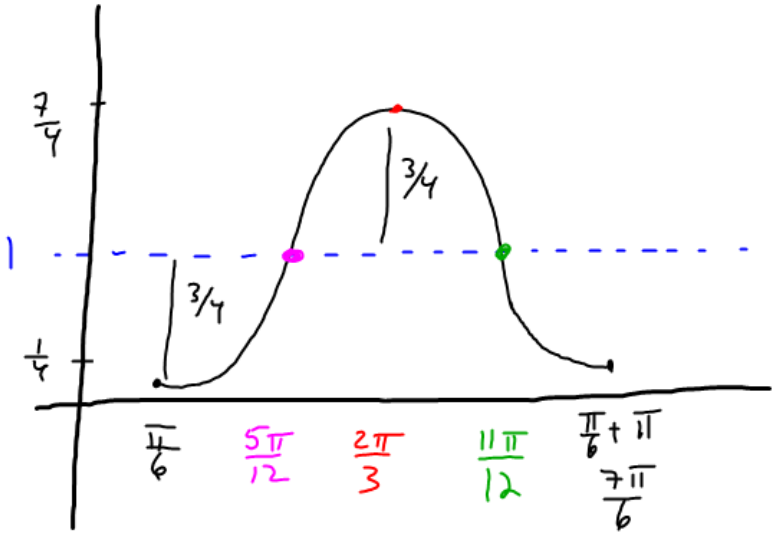
$\frac{\pi}{12} + \frac{5\pi}{12} = \frac{6\pi}{12} \cdot \frac{1}{2} = \frac{3\pi}{12} = \frac{\pi}{4}$

My Steps

1. Sketch the curve
2. Put in midline d-value
3. Find the period
   $P = \frac{2\pi}{b}$
   $P = \frac{2\pi}{3}$
4. Find the shift, put in start point
   left $\frac{3\pi}{12} = \frac{\pi}{4}$
5. Find endpoint: start point + period
6. Average to find x-coord.
7. Put in axes, Find y-coord ± midline ± a-term

Example

$$y = \frac{-3}{4}\cos\left[2\left(x - \frac{\pi}{6}\right)\right] + 1$$



$$P = \frac{2\pi}{6} \Rightarrow \frac{2\pi}{2} = \pi$$

$$\frac{\pi}{6} + \frac{2\pi}{6} = \frac{8\pi}{6} \cdot \frac{1}{2} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$\frac{\pi}{6} + \frac{2\pi}{3} = \frac{\pi}{6} + \frac{4\pi}{6} = \frac{5\pi}{6} \cdot \frac{1}{2} = \frac{5\pi}{12}$$

$$\frac{2\pi}{3} + \frac{7\pi}{6} = \frac{4\pi}{6} + \frac{7\pi}{6} = \frac{11\pi}{6} \cdot \frac{1}{2} = \frac{11\pi}{12}$$

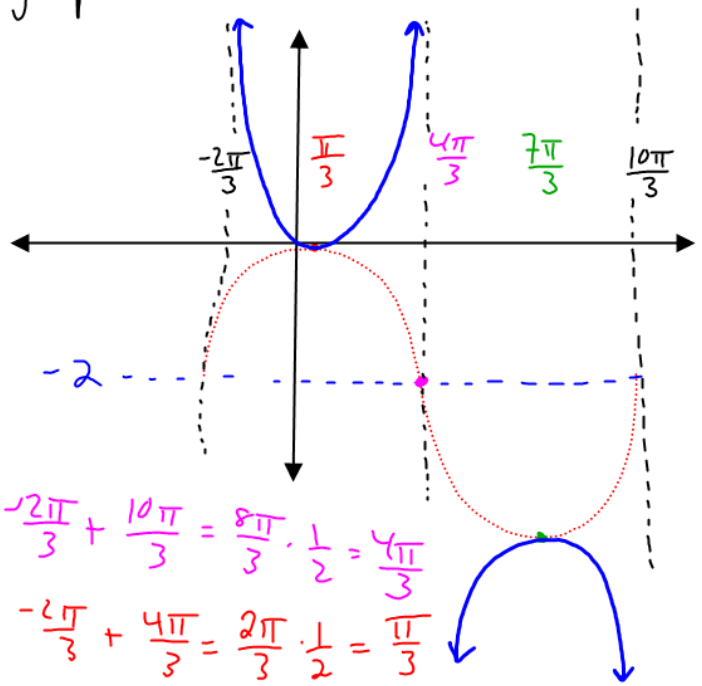
My steps

1. Draw curve first
2. Put in midline
3. Find period
4. Find the horz. shift to get the starting endpoint
5. Find the ending endpoint using the start point plus the period
6. Find x-coord. by avg.
7. Put in axes and find y-coord. using the a-term

Secant and Cosecant

$y = 2\csc\left[\frac{1}{2}\left(x + \frac{2\pi}{3}\right)\right] - 2 \rightarrow y = 2\sin\left[\frac{1}{2}\left(x + \frac{2\pi}{3}\right)\right] - 2$

To graph secant and cosecant, graph their reciprocal function. For sec(x) graph cos(x), for csc(x) graph sin(x). Put in vertical asymptotes where the reciprocal graph crosses the mid line.



$-\frac{2\pi}{3} + \frac{10\pi}{3} = \frac{8\pi}{3} \cdot \frac{1}{2} = \frac{4\pi}{3}$

$-\frac{2\pi}{3} + \frac{4\pi}{3} = \frac{2\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{3}$

$\frac{4\pi}{3} + \frac{10\pi}{3} = \frac{14\pi}{3} \cdot \frac{1}{2} = \frac{7\pi}{3}$

$P = \frac{2\pi}{\frac{1}{2}} = 4\pi$

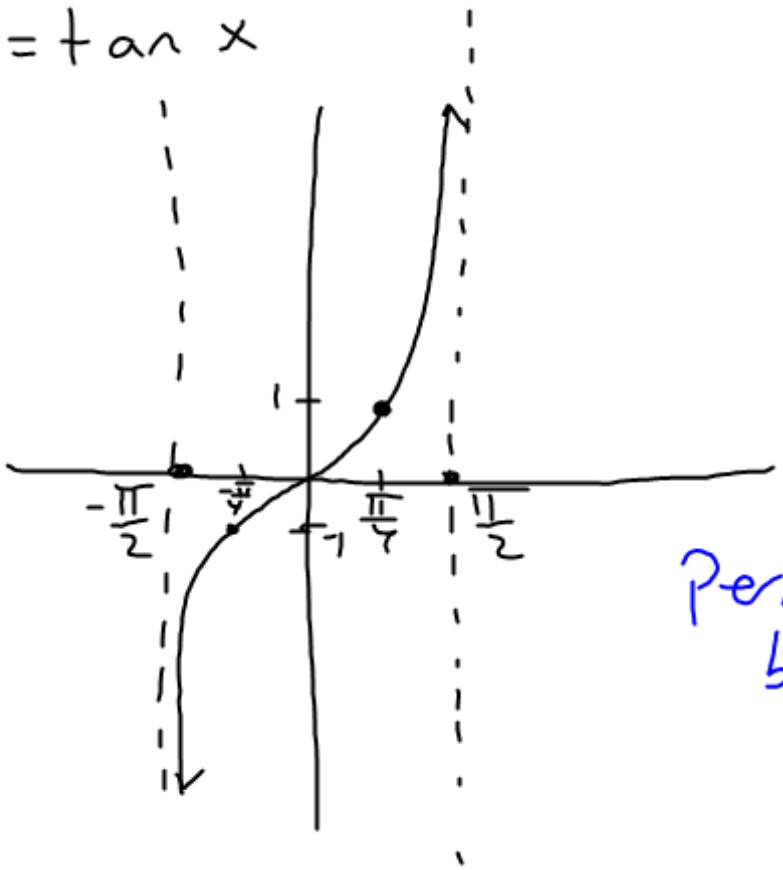
Start: $-\frac{2\pi}{3}$

End: $-\frac{2\pi}{3} + 4\pi$

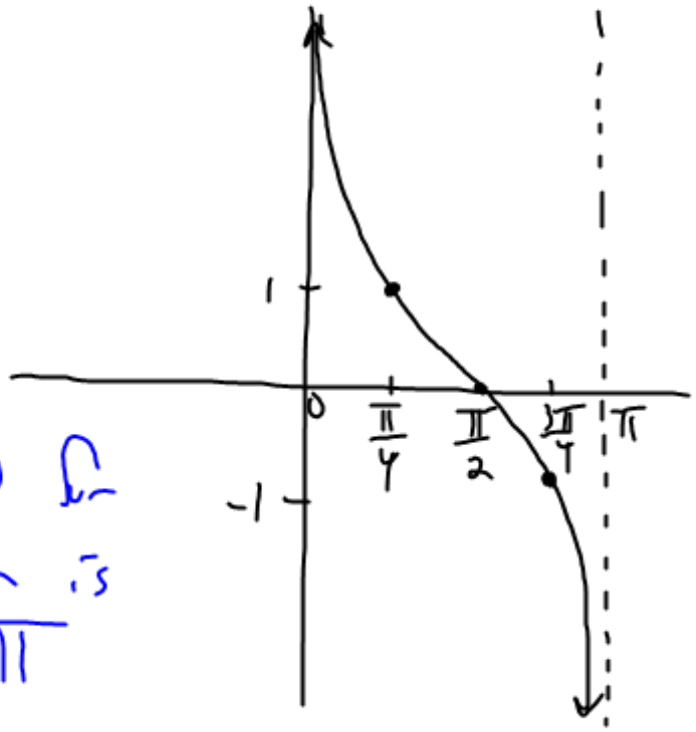
$= \frac{10\pi}{3}$

tangent & cotangent

$y = \tan x$



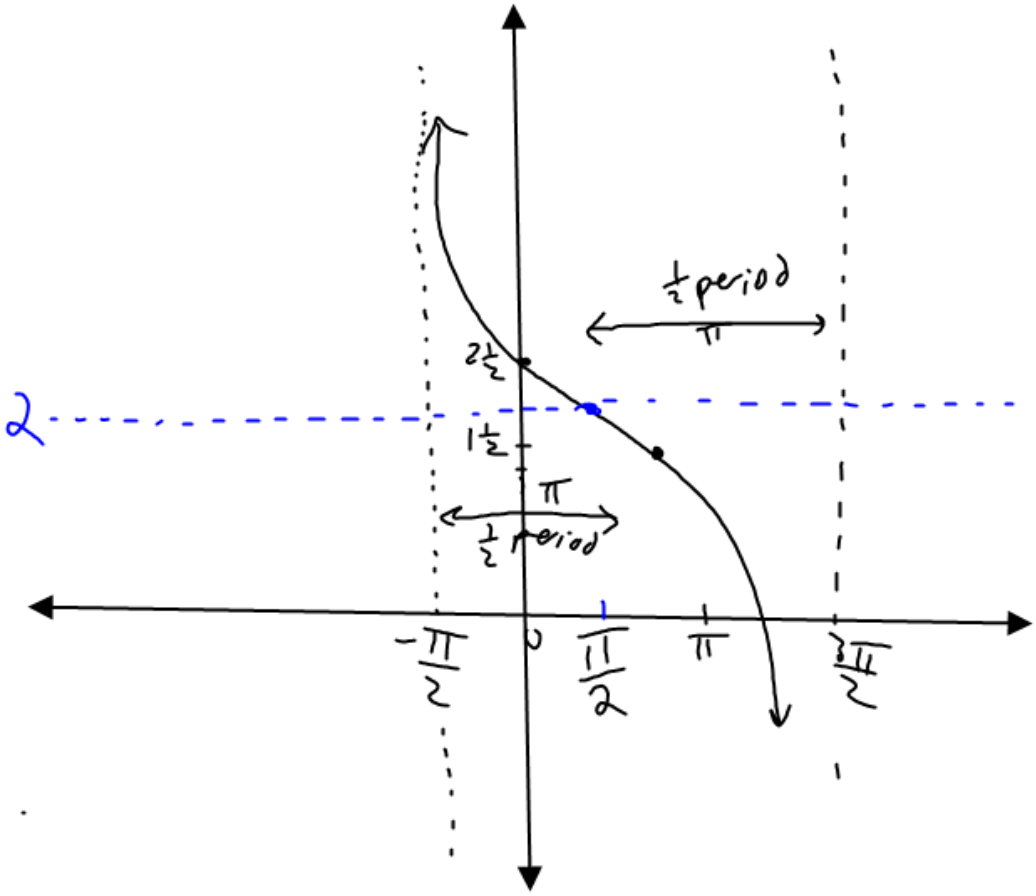
$y = \cot x$



Period for both is π

Example

$$y = -\frac{1}{2}\tan\left[\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right] + 2$$



① sketch curve

② Find period

$P = \frac{\pi}{b}$ $P = \frac{\pi}{\frac{1}{2}} = 2\pi$

③ Find the shift using the center

④ start point is the center $-\frac{1}{2}$ Period

⑤ end point is the center + $\frac{1}{2}$ period

⑥ Put in axes

⑦ $\frac{1}{2}$-way between center and asymptotes Find y-coord. by doing midline plus and minus a-term

$\cos\left(2x - \frac{\pi}{2}\right)$ ⟶ ① $\cos\left(\frac{2x}{2} - \frac{\frac{\pi}{2}}{2}\right)$

$\cos\left[2\left(x - \frac{\pi}{4}\right)\right]$

What is the horz. shift?

$\frac{\pi}{4}$

② $0 \le 2x - \frac{\pi}{2} \le 2\pi$

$+\frac{\pi}{2} \quad +\frac{\pi}{2} \quad +\frac{\pi}{2}$

$\frac{\frac{\pi}{2}}{2} \le \frac{2x}{2} \le \frac{\frac{5\pi}{2}}{2}$

$\frac{\pi}{4} \le x \le \frac{5\pi}{4}$

HW

$$y = \cot\left(\frac{1}{2}x + \frac{\pi}{8}\right) - 2$$

$$y = -2\sin\left[\frac{\pi}{3}(x-2)\right] + 1$$

$$y = 3\sec\left(\pi x - \frac{\pi}{3}\right)$$

$$y = 4 + 2\cos\left(2x - \frac{2\pi}{3}\right)$$

$$y = 2\tan\left[\frac{1}{2}\left(x + \frac{\pi}{4}\right)\right] + 1$$